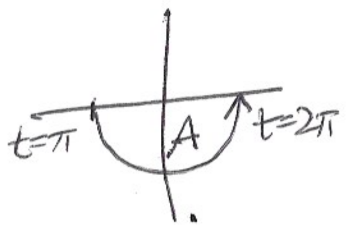
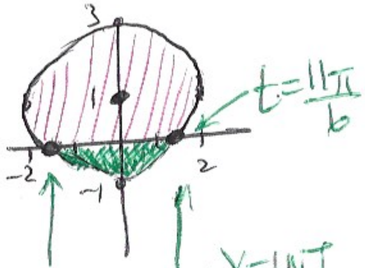


$$-A=\int_{\pi}^{2\pi} a\sin t\cdot -a\sin t\,dt$$

$$A=a^2\int_{\pi}^{2\pi}\sin^2 t\,dt = \frac{1}{2}a^2\left(t-\frac{1}{2}\sin 2t\right)\Big|_{\pi}^{2\pi} = \frac{1}{2}a^2(2\pi-\pi)=\frac{1}{2}\pi a$$

TOTAL $\frac{1}{2}\pi a^2+\frac{1}{2}\pi a^2=\pi a^2$

FIND AREA OF PORTION OF CIRCLE OF RADIUS 2 WITH CENTER @ (0,1) THAT IS ABOVE X-AXIS



$$A=-\int_{7\pi/6}^{11\pi/6}(1+2\sin t)(-2\sin t)\,dt \qquad x=2\cos t,\quad y=1+2\sin t$$

$t=\frac{7\pi}{6}$

X-INT

$y=0$

$1+2\sin t=0$

$\sin t=-\frac{1}{2}$

$t=\frac{7\pi}{6},\ \frac{11\pi}{6}$

$$=2\int_{7\pi/6}^{11\pi/6}(\sin t+2\sin^2 t)\,dt$$

$$=2\Big(-\cos t+\boxed{\cancel{1-\cos 2t}}\overset{t-\frac{1}{2}\sin 2t}{}\Big)\Big|_{7\pi/6}^{11\pi/6}$$

IGNORE IN

$$\cancel{=2\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\right)}$$

$$\cancel{=-\sqrt{3}-1-\sqrt{3}+1=-2\sqrt{3}}$$

$$=2\left(-\cos t+t-\frac{1}{2}\sin 2t\right)\Big|_{7\pi/6}^{11\pi/6}$$

$$=2\Big(-\cos\frac{11\pi}{6}+\frac{11\pi}{6}-\frac{1}{2}\sin\frac{11\pi}{3} \quad 3\tfrac{2}{3}\pi$$

$$+\cos\frac{7\pi}{6}-\frac{7\pi}{6}+\frac{1}{2}\sin\frac{7\pi}{3}\Big) \quad 2\tfrac{1}{3}\pi$$

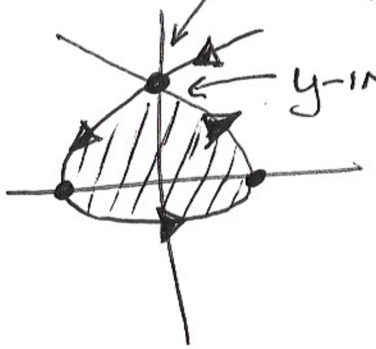
$$=2\left(-\frac{\sqrt{3}}{2}-\cancel{\frac{\sqrt{3}}{2}}+\frac{4\pi}{6}+\frac{1}{2}\cancel{\frac{\sqrt{3}}{2}}+\frac{1}{2}\frac{\sqrt{3}}{2}\right)=\frac{4\pi}{3}-\sqrt{3} \text{ =GREEN AREA}$$

$$A=4\pi-\left(\frac{4\pi}{3}-\sqrt{3}\right)$$

$$=\frac{8\pi}{3}+\sqrt{3}$$

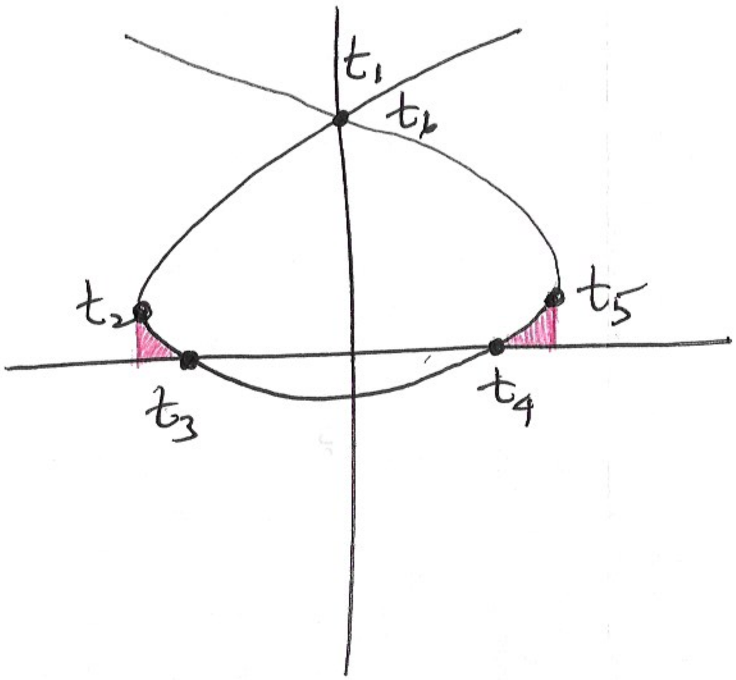
FIND THE AREA INSIDE THE LOOP OF $x = 4t - t^3$

$y = t^2 - 1$



$$y\,dx = (t^2-1)(4-3t^2)\,dt$$

$$\int_{t_2}^{t_1} (t^2-1)(4-3t^2)\,dt$$



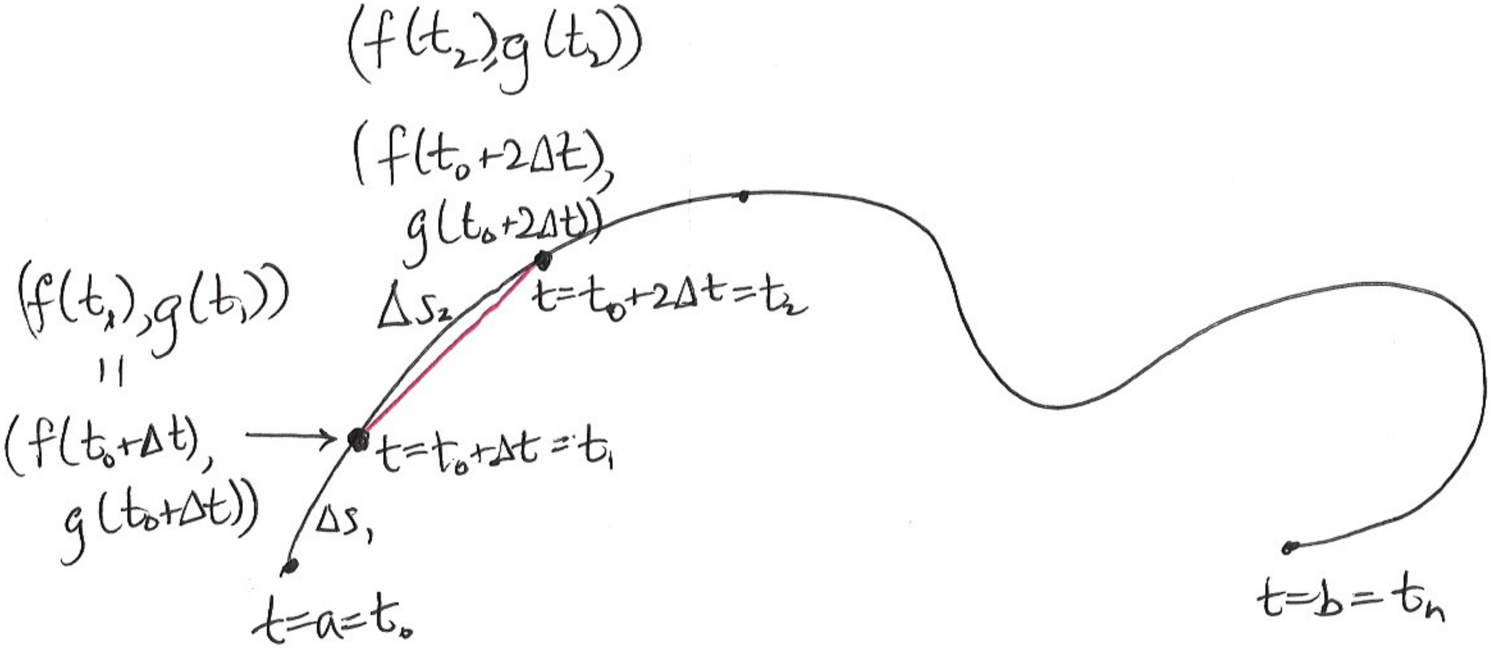
$$- \int_{t_2}^{t_3} (t^2-1)(4-3t^2)\,dt$$

$$- \int_{t_3}^{t_4} \quad ''$$

$$- \int_{t_4}^{t_5}$$

$$+ \int_{t_6}^{t_5}$$

ARCLENGTH OF $x = f(t)$, $y = g(t)$, $t \in [a,b]$ } IF THE CURVE IS ONLY TRACED ONCE WITHOUT "DOUBLEBACK"



$$\Delta s_2 \approx \sqrt{(f(t_2) - f(t_1))^2 + (g(t_2) - g(t_1))^2}$$

$$\Delta s_i \approx \sqrt{(f(t_i) - f(t_{i-1}))^2 + (g(t_i) - g(t_{i-1}))^2}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \Delta s_i = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{(f(t_i) - f(t_{i-1}))^2 + (g(t_i) - g(t_{i-1}))^2}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\left(\frac{f(t_i)-f(t_{i-1})}{\underbrace{t_i - t_{i-1}}_{=\Delta t}}\right)^2 + \left(\frac{g(t_i)-g(t_{i-1})}{\underbrace{t_i - t_{i-1}}_{=\Delta t}}\right)^2}\ \Delta t$$

BY MVT (1A)

$= f'(t_i^*)$ $\qquad\qquad$ $= g'(t_i^\star)$

FOR SOME $t_i^* \in [t_{i-1}, t_i]$ $\qquad$ FOR SOME $t_i^\star \in [t_{i-1}, t_i]$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{(f'(t_i^*))^2 + (g'(t_i^\star))^2}\ \Delta t$$

$$s = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

IF f $+g$ ~~IS~~ ARE CONT. ON $[a,b]$

FIND ARCLENGTH OF $x = 3\cos t - \cos 3t$

$y = 3\sin t - \sin 3t$

$t \in [0, \pi]$

$$\int_0^{\pi} \sqrt{(-3\sin t + 3\sin 3t)^2 + (3\cos t - 3\cos 3t)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{\begin{matrix} 9\sin^2 t - 18\sin t\sin 3t + 9\sin^2 3t \\ +9\cos^2 t - 18\cos t\cos 3t + 9\cos^2 3t \end{matrix}}\, dt$$

$$= \int_0^{\pi} \sqrt{18 - 18(\sin t\sin 3t + \cos t\cos 3t)}\, dt$$

$$= \int_0^{\pi} \sqrt{18 - 18(\cos(3t - t))}\, dt$$

$$= \int_0^{\pi} 3\sqrt{2}\sqrt{1 - \cos 2t}\, dt$$

$\cos 2t = 1 - 2\sin^2 t$

$1 - \cos 2t = 2\sin^2 t$

$$= \int_0^{\pi} 3\sqrt{2}\sqrt{2\sin^2 t}\, dt = 6\int_0^{\pi} |\sin t|\, dt = 6\int_0^{\pi} \sin t\, dt = -6\cos t\Big|_0^{\pi}$$

$= -6(-1-1)$

$= 12$